

# MATH 7 Review (2016 standards)

## 7.1a,b NEGATIVE EXPONENTS FOR POWERS OF TEN

### Negative Exponents

$$5^{-2} = \frac{1}{5^{-2}} = \frac{1}{25}$$

**Negative exponents** represent numbers between 0 and 1.  $10^{-3} = \frac{1}{10^3} =$

0.001 = one thousandths

What does  $10^{-2}$  represent?  
name for hundredths)

Answer: 0.01 or 1% (Percent is another

**Scientific notation** used for **very large** and **very small** numbers.

Scientific notation has two parts – a **decimal between 1 and 10** (examples: 1.0, 1.456, 6.4, 9.99) and a **power of ten** ( $10^5$ ,  $10^{-3}$ )

To write 145,600 in scientific notation, move the decimal point over until you have a decimal between 1 and 10.

$145,600 = 145600.00 = 1.456 \times 10^5$  (the decimal point was moved 5 places to the left)

Write 0.000345 in scientific notation – move the **decimal point right** until you have a number between 1 and 10.

Answer:  $3.45 \times 10^{-4}$  (the decimal point was moved right 4 places to get 3.45)

### Scientific Notation

**450,000**  
5  
move the decimal point left so you end up with a number between 0 and 1.  
 $4.5 \times 10^5$

**.000045**  
5  
move the decimal point right so you end up with a number between 0 and 1.  
 $4.5 \times 10^{-5}$

**.0000455**  $4.55 \times 10^{-5}$

**NOT**  ~~$45.5 \times 10^{-4}$~~  45.5 is not between 0 and 1

## RATIONAL NUMBERS

**Integers** are positive and negative whole numbers (and zero)

**Rational numbers** – all numbers that can be written as fractions with denominators not zero. Examples  $\sqrt{25}$ ,  $\frac{1}{4}$ , -2.3, 82, 75%,  $4.\overline{59}$ .

**Proper fraction** – numerator less than denominator -  $\frac{1}{2}$ ,  $\frac{7}{8}$

**Improper fraction** – numerator equal or greater than denominator –  $\frac{12}{7}$ ,  $\frac{4}{4}$ .

Improper fractions can be written as **mixed numbers** -  $3\frac{5}{8}$ ,  $2\frac{1}{4}$

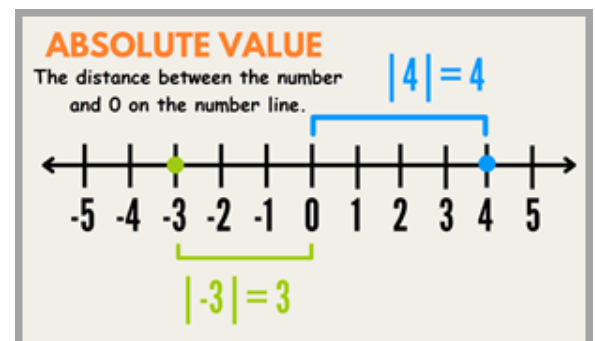
A **perfect square** is a whole number whose square root is an integer: 4, 9, 16, 25, 36 etc

The symbol  $\sqrt{\quad}$  represents a square root.

$\sqrt{36} = 6$  means the square root of 36 = 6 ( $6 \times 6 = 36$ )

$\sqrt{81} = ?$  Answer: 9 ( $9 \times 9 = 81$ )

The **absolute value** of a number is the distance of that number from zero on the number line.



## 7.2 SOLVE PRACTICAL PROBLEMS USING RATIONAL NUMBERS

**Rational numbers** - All numbers that can be expressed as fractions in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers and  $b$  does not equal zero. A rational number can be written as a decimal or as a repeating decimal (line over repeating digits).

**Proper fraction** – numerator less than denominator. example  $\frac{3}{4}$ .

**Improper fraction** - numerator is equal to or greater than the denominator. example  $\frac{4}{3}$

**Mixed number** – improper fraction can be written as a mixed number. example  $\frac{4}{3} = 1\frac{1}{3}$

Students will solve addition, subtraction, multiplication, and division problems with rational numbers.

## 7.3 PROBLEMS USING PROPORTIONAL REASONING

A **proportion** (introduced in grade 6) is an equation which states that two ratios are equal. A proportion can be written as  $\frac{a}{b} = \frac{c}{d}$      $a : b = c : d$      $a$  is to  $b$  as  $c$  is to  $d$ .

**Equivalent ratios** – multiply each value in ratio by same number. 5 : 4 is equivalent to 10 : 8 and 20 : 16.

To solve a proportion with a missing value ( $y$ ),  $\frac{2}{3} = \frac{y}{9}$ , **cross multiply** like this:  $2 \times 9 = 3y$ , so  $18 = 3y$  so  $6 = y$

A recipe calls for 3 eggs for every 6 cups of flour. How many eggs would you need with 2 cups of flour?

$$\frac{3}{6} = \frac{y}{2} \quad 3 \times 2 = 6y \quad 6 = 6y \quad y = 1 \text{ egg}$$

**Rate** - ratio that compares two quantities measured in **different units**.

**Unit rate** – has 1 as denominator. example miles/hour A bike might travel 10 miles/one hour. Rate is  $\frac{10}{1}$ .

How far would the car travel in 4 hours?  $\frac{10}{1} = \frac{y}{4}$  Cross multiply to solve.  $10 \times 4 = 1y$   $y = 40$  miles

Proportions can be used to convert **length, weight (mass), and volume (capacity)** within and between **measurement systems**.

-**Length**: between feet and miles; miles and kilometers example approx. 1 mile = 1.6km

In miles, how long is a 10km race?  $\frac{1}{1.6} = \frac{y}{10}$  ->  $1 \times 10 = 1.6y$  ->  $10 = 1.6y$  ->  $10 \div 1.6 = y$  ->  $10 \text{ km} = 6.25 \text{ mi}$ .

-**Weight**: between ounces and pounds; pounds and kilograms

-**Volume**: between cups and fluid ounces; gallons and liters

- **Percent** – ratio in which denominator is 100. To turn a **fraction into a percent** use this:  $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$

Turn  $\frac{3}{4}$  into a percent:  $\frac{3}{4} = \frac{y}{100}$  ->  $3 \times 100 = 4y$  ->  $300 = 4y$  ->  $y = 300 \div 4$  ->  $y = 75$  ->  $\frac{3}{4} = 75\%$

## 7.4 VOLUME AND SURFACE AREA OF RECTANGULAR PRISMS AND CYLINDERS

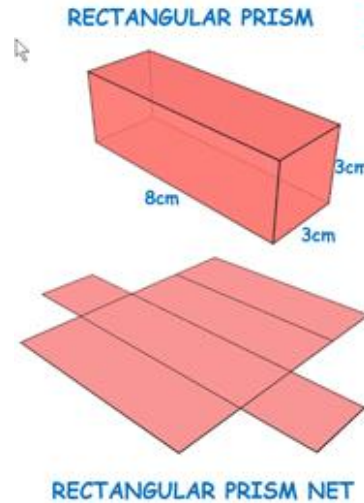
**Polyhedron** is a solid figure whose **faces** are all **polygons**.

**Rectangular prism** is a polyhedron in which all **six faces** are **rectangles** - 8 vertices and 12 edges

A **face** is any flat surface of a solid figure.

The **surface area** of a prism is the sum of the areas of **all 6 faces** and is measured in **square units**

The **volume** of a three-dimensional figure is a measure of **capacity** and is measured in **cubic units**.



**SURFACE AREA** - add up the surface areas of all 6 faces  
 $9 + 9 + 24 + 24 + 24 + 24 = 144\text{cm}^2$   
*don't forget squared*

**VOLUME** - volume of base  $\times$  height  
 If we call a smaller face the base -  
 $3 \times 3 = 9$  (base volume)  
 $9 \times 8$  (height) = 72  
**VOLUME =  $72\text{cm}^3$**   
*don't forget cubed*

**Cylinder** - bases joined by a curved surface

Know how to find surface area and volume of cylinders and rectangular prisms.

Find the **surface area** and **volume** of a  $3 \times 3 \times 8$  rectangular prism (see previous page)

Find the **volume** and **surface area** of a the **cylinder** with a radius of 2cm and height of 4cm. (see above)  $\pi = 3.14$

**CYLINDER**

$r = 2\text{cm}$   
 $h = 4\text{cm}$

**SURFACE AREA**  
 area of 2 circles + area of rectangle (see net)  
 $A = \pi r^2 = 12.57$   
 area of 2 circles = approx **25**  
 area of rectangle = 50 (see below)  
**total surface area = approx  $75\text{cm}^2$**

**VOLUME**  
 height  $\times$  area of base ( $\pi r^2$ )  
 $4 \times 12.57 = 50.28\text{cm}^3$

**CYLINDER NET**

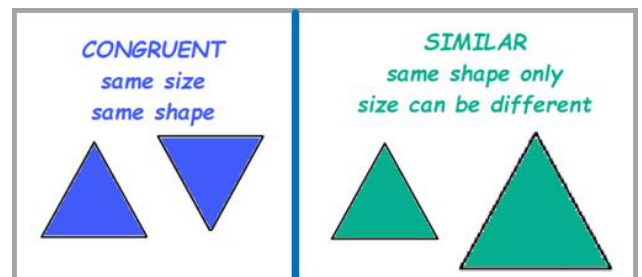
$l = 2\pi r =$

**AREA =  $l \times h$**   
 The **rectangle** is made by unrolling the cylinder, so the rectangle's length equals the circumference of the cylinder.  
 $l = 2\pi r = 12.57$   
 $A = 4 \times 12.57 = \text{approx } 50$

## 7.5 CORRESPONDING SIDES AND ANGLES OF SIMILAR QUADRILATERALS AND TRIANGLES

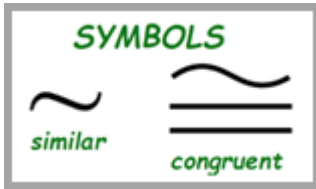
**Similar polygons** – angles are congruent, sides are proportional but not necessarily congruent.

**Conguent polygons** – angles and sides are congruent, same size and shape.



**Congruent polygons are similar**, but the reverse is not necessarily true.

In the similar triangles to the right, what length are the missing sides, EF and AC (use the ratio of  $\frac{1}{2}$ )



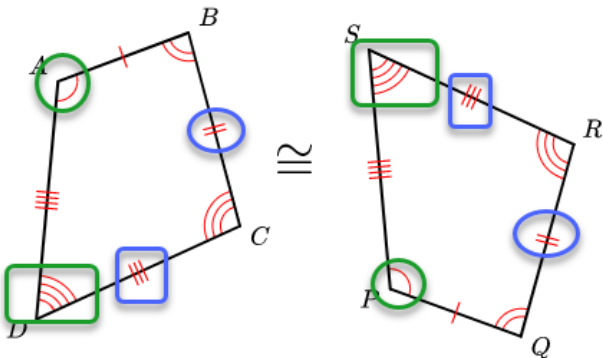
**These triangles are similar**

Sides are proportional  
ratio 1 : 2

$$\frac{14}{28} = \frac{12}{EF} = \frac{AC}{50}$$

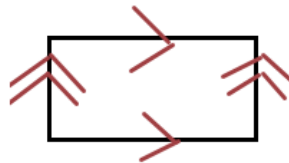
$\overline{EF} = 24$      $\overline{AC} = 25$

**MARKING CONGRUENT ANGLES AND SIDES**



Congruent sides are marked with same number of hash marks.  
Congruent angles are marked with equal number of curves

**MARKING PARALLEL SIDES**

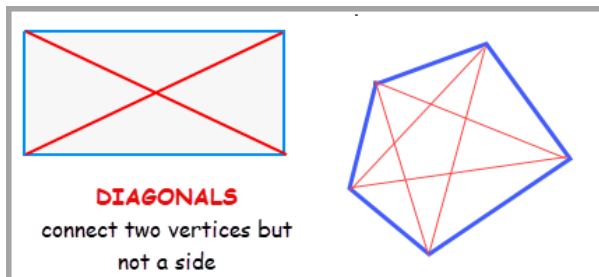


Equal numbers of arrows indicate that sides are parallel.

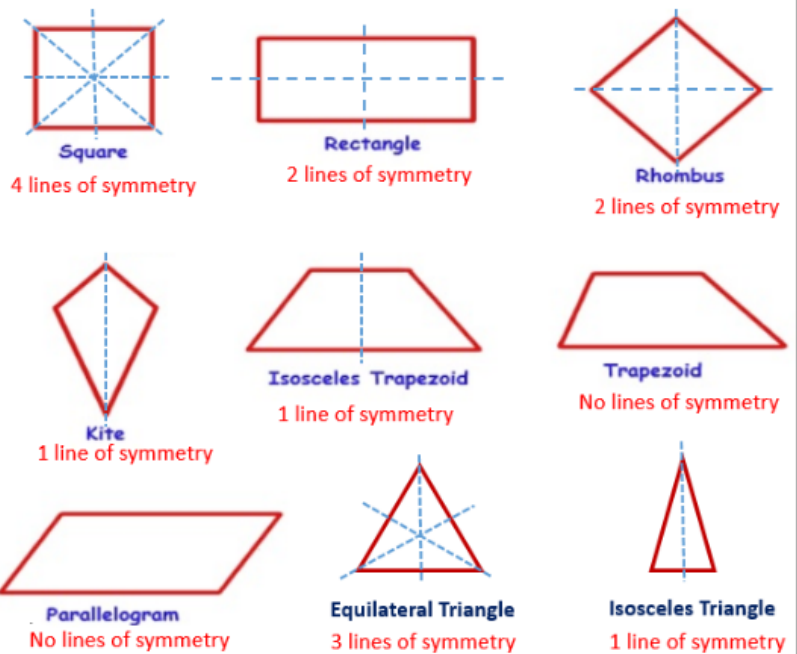
**7.6 WORKING WITH QUADRILATERALS**

**Polygon** – a closed plane figure with at least 3 sides that don't cross

**Quadrilateral** -a polygon with four sides.



**Lines of Symmetry**



**Bisect** – divide into two equal parts.

**Line of symmetry** – divides a figure into two congruent parts, each a mirror image of the other.

**Parallelogram** - a quadrilateral with both pairs of opposite sides parallel.

**Rectangle** - a quadrilateral with four right angles.

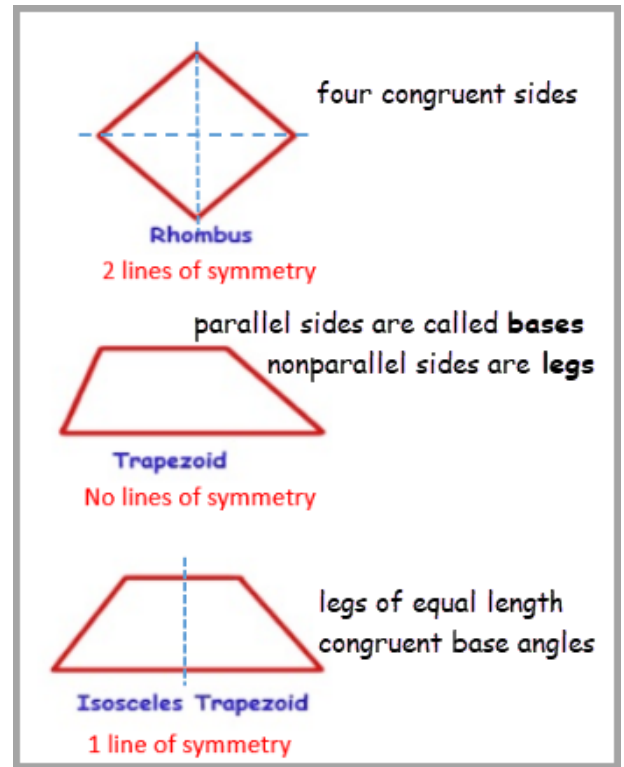
**Square** – polygon with four congruent sides and four right angles.

**Rhombus** - a quadrilateral with four congruent sides.

**Trapezoid** - a quadrilateral with exactly one pair of parallel sides. Parallel sides are called **bases**. Nonparallel sides are called **legs**.

**Isosceles trapezoid** - has legs of equal length and congruent base angles.

The sum of the measures of the **interior angles** of a quadrilateral is  $360^\circ$ .



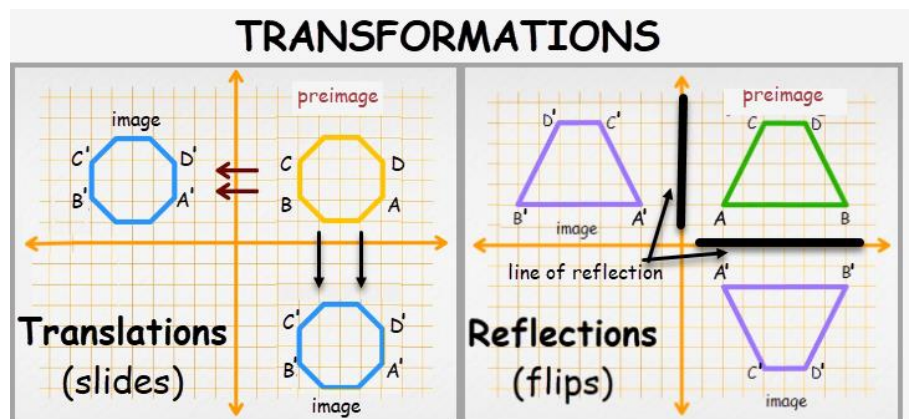
## 7.7 TRANSLATIONS AND REFLECTIONS OF RIGHT TRIANGLES OR RECTANGLES IN THE COORDINATE PLANE

**Transformation** – changes the **preimage** in **size**, **shape** or **position**. New figure called **image**.

**Translations** and **reflections** change only the **position** of the preimage, **not** the **size** or **shape**.

**Translation** – preimage slides to different position.

**Reflection** – preimage is **reflected** over a **line of reflection**.

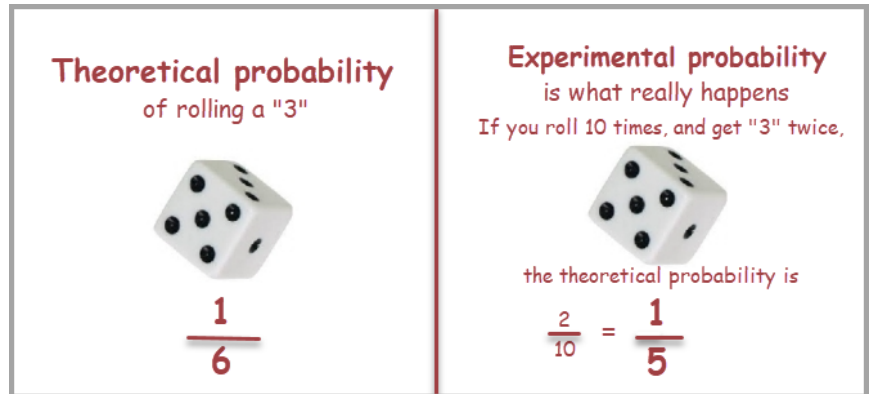


## 7.8 THEORETICAL AND EXPERIMENTAL PROBABILITIES OF AN EVENT

The **probability** of an event occurring is a **ratio between 0 and 1**.

- A probability of 0 means the event will never occur.
- A probability of 1 means the event will always occur.

The **theoretical probability** of an event is the **expected probability** and can be determined with a **ratio**.



The **experimental probability** of an event is determined by carrying out a simulation or an **experiment**.

The **more trials**, the closer the **experimental probability** will be to the **theoretical**. In the example above, if the die was rolled 100 times, the experimental probability would likely be nearer to  $\frac{1}{6}$ .

## 7.9 FOCUS ON HISTOGRAMS; COMPARE WITH STEM-AND-LEAF PLOTS, LINE PLOTS, AND CIRCLE GRAPHS

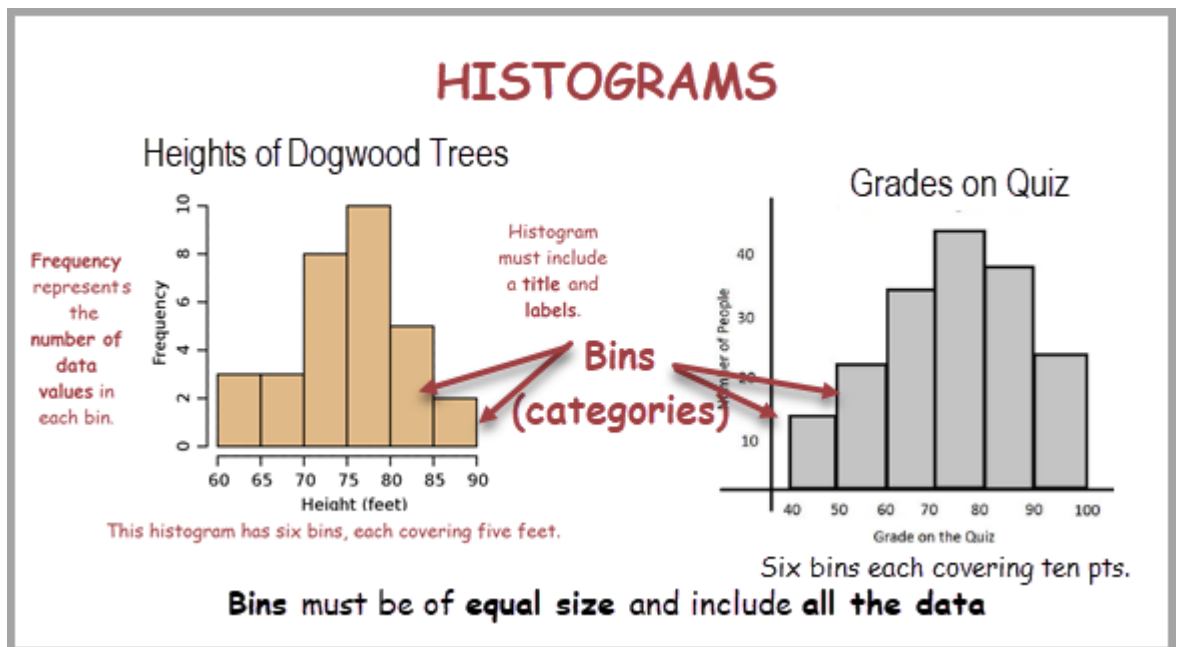
A **histogram** is a form of **bar graph** in which the **categories** are **consecutive** and **equal intervals**. The length or **height** of each bar is determined by the **number of data elements** (frequency) falling into a particular interval.

**Bins (categories)** on x-axis must be of **equal size** and must include **all the data**.

The **frequency** (**number of data points**) on y-axis.

x-axis and y-axis can be switched so that bars are **horizontal**.

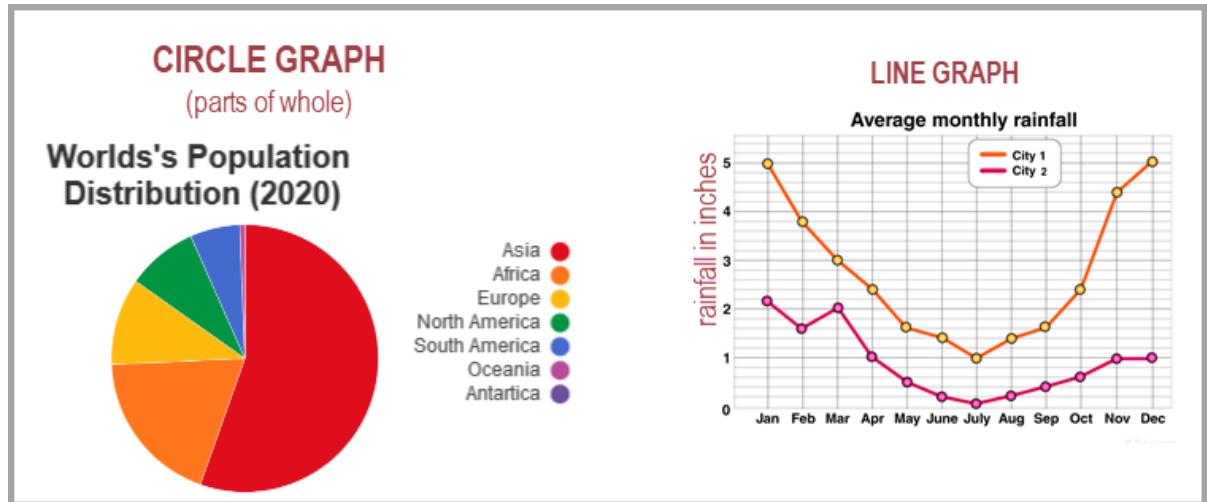
Can be used with **categorical** data or **numerical** data.



The type of graph used depends on the data and what the graph is intended to show.

**Line plots** are good for showing the **spread of data** and might be more useful when there are **extreme** high or low values.

**Circle graphs** are used to show a relationship between the **parts and the whole**.



## 7.10 PROPORTIONAL VS. ADDITIVE RELATIONSHIPS, GRAPHS OF LINES, SLOPES, Y-INTERCEPTS

**Slope** may also represent the **unit rate** of a **proportional relationship** between two quantities, also referred to as the **constant of proportionality** or the **constant ratio** of **y** to **x**.

Relationship represented as  $y = mx$ , where **m** is the **slope**.

$$\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$$

The equation representing this proportional relationship of **y** to **x** is  $y = \frac{1}{2}x$  or  $y = 0.5x$

The **slope** of a line representing this relationship is  $\frac{1}{2}$  or **.5**

Slope represents the rate of change of a line.

$$\frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}}$$

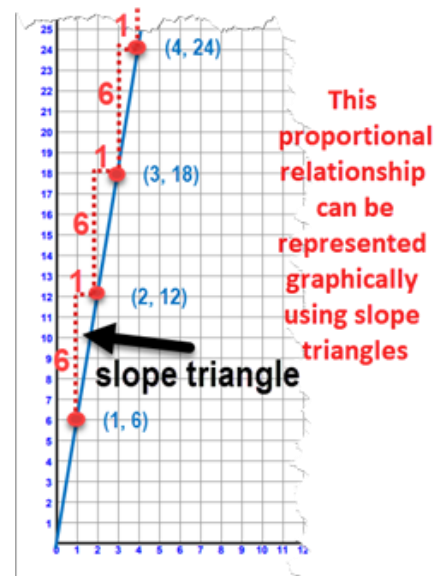
The graph of the line representing a **proportional relationship** will include the **origin (0, 0)**.

**Practical Problem:** John runs 1 mile every 6 minutes. In this example, he never tires. This table represents the relationship.

x (miles)	1	2	3	4	5
y (minutes)	6	12	18	24	30

$$\frac{y}{x} = \frac{6}{1} = \frac{12}{2} = \frac{18}{3} = \frac{24}{4} = \frac{30}{5} = 6$$

This proportional relationship can be represented by the equation  $y = 6x$ .



### MULTIPLICATIVE

x	y
1	2
2	4
3	6
4	8

### ADDITIVE

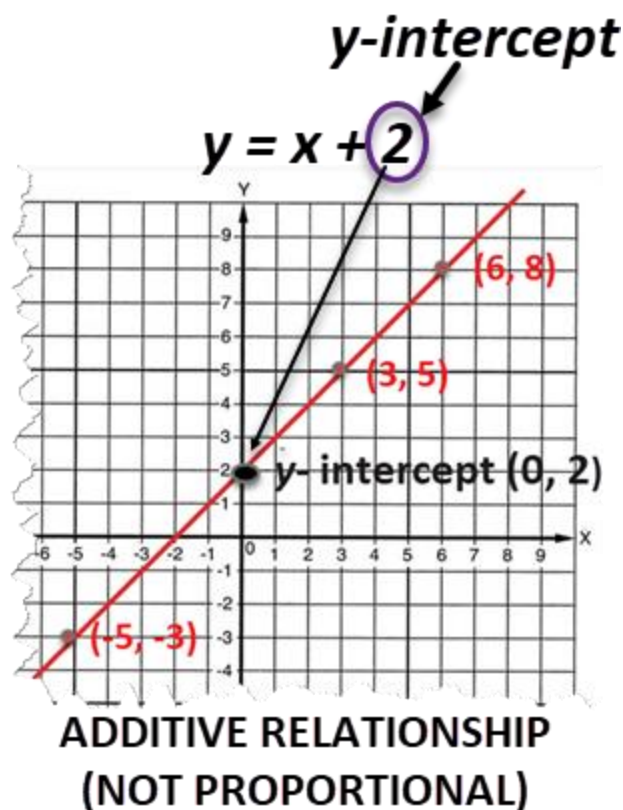
x	y
1	3
2	4
3	5
4	6

The relationships above were **multiplicative**, but relationships can also be **additive**.

An **additive** relationship is **not proportional** and its graph does **not pass through (0, 0)**.

The equation for the additive relationship is  $y = x + 2$

The **slope** of this line is **1**. If this relationship was proportional, it would pass through the origin (0, 0). Instead, note that it passes through (0, 2) which is called the **y-intercept**.



## 7.11 EVALUATING ALGEBRAIC EXPRESSIONS

Evaluate an algebraic expression  $4a - (2 + 3)b$  given values  $a = 2$   $b = 3$

- 1) Replace the variables with given numbers  $4(2) - (2 + 3)(3)$
- 2) simplify using order of operations  $8 - 5 \times 3; 8 - 15; -7$

### Review of Order of Operations

**Grouping symbols** ( ), [ ], etc innermost first

**Exponents**

**Multiply** and/or **divide**, left to right

**Add** and/or **subtract**, left to right

### Review of Properties

- **Commutative** property of **addition**:  $a + b = b + a$ .
- **Commutative** property of **multiplication**:  $a \cdot b = b \cdot a$ .
- **Associative** property of **addition**:  $(a + b) + c = a + (b + c)$ .
- **Associative** property of **multiplication**:  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ .

Subtraction and division are neither commutative nor associative.

- **Distributive** property (over addition/subtraction):  $a \cdot (b + c) = a \cdot b + a \cdot c$  and  $a \cdot (b - c) = a \cdot b - a \cdot c$ .



- **Identity** property of **addition** (additive identity property):  $a + 0 = a$  and  $0 + a = a$ .
- **Identity** property of **multiplication** (multiplicative identity property):  $a \cdot 1 = a$  and  $1 \cdot a = a$ .
- **Inverse** property of **addition** (additive inverse property):  $a + (-a) = 0$  and  $(-a) + a = 0$ .
- **Inverse** property of **multiplication** (multiplicative inverse property):  $a \cdot \frac{1}{a} = 1$  and  $\frac{1}{a} \cdot a = 1$ .
- **Multiplicative** property of **zero**:  $a \cdot 0 = 0$  and  $0 \cdot a = 0$ .

Division by zero is not a possible mathematical operation. It is undefined.

- **Substitution** property: If  $a = b$ , then  $b$  can be substituted for  $a$  in any expression, equation, or inequality.

## 7.12 Solving two-step linear equations in one variable

An **equation**: states that the mathematical expression on the left of the equal sign is equal to the expression on the right so **expression = expression**  $2x = 4^2$

The Solution to an equation is what makes it true -  $2x = 4^2$  Solution:  $x = 8$

An **expression** itself does not include an equal sign

A **variable expression** contains a variable:  $5y$ ;

An algebraic expression is a variable contains a variable:  $5y + 3$

More review of properties included here – see 7.12

## 7.13 Solving one- and two-step linear inequalities in one variable

**SOLVING INEQUALITIES** When dividing by a **negative number**, change the **direction of the sign**

$$2x + 1 < 9$$

$$2x < 8$$

$$x < 4$$

$$1 - 2x < 9$$

$$-2x < 8$$

$$2 > -4$$

**Inequalities on a number line**

$n > -1$

$n \leq 3$

$-1 < n < 3$

When both expressions of an inequality are multiplied or divided by a **negative number**, the inequality **symbol reverses** (e.g.,  $-2x < 6$  is equivalent to  $x > -3$ ).